

# GENERAL SCALE INTERPOLATION BASED ON FINE-GRAINED ISOPHOTE MODEL WITH CONSISTENCY CONSTRAINT

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## ABSTRACT

In this paper, we propose a fine-grained isophote model with consistency constraint to characterize the piecewise-stationarity of image signals. According to this model, we present a novel interpolation algorithm. In this model, the displacement coefficient is used to model the isophote. Then fine-grained pixel intensity information is introduced to correct the displacement calculation and make the isophote estimation more robust. In order to handle the piecewise-stationarity, we force the isophote direction consistent in the local window when an interpolated line is piecewise-stationary. The proposed algorithm can accommodate the general scale enlargement. Experimental results demonstrate that the proposed approach achieves better performances in both objective and subjective quality assessment.

*Index Terms*— General scale, Isophote-based, Interpolation

## 1. INTRODUCTION

Interpolation is a general and economic technique for the image enlargement. It generates high resolution image by utilizing low resolution image information with some prior knowledge. Conventional interpolation methods, such as Bilinear and Bicubic, interpolate an image by convolving pixels with a fixed kernel. These methods do not adapt to local structure of images therefore artifacts, such as blurring and ringing, occur in the frontier of different regions.

In order to overcome the above deficiency, edge-directed interpolation methods are proposed. These methods pay attention to edge modelling and adapt to local structure, such as New edge-directed interpolation (NEDI) [1] and Soft-decision adaptive interpolation (SAI) [2]. NEDI uses the low-resolution (LR) image to estimate the high resolution (HR) covariances by a least square problem and estimates HR pixels by their neighbor LR pixels using corresponding covariances. SAI extends the framework of NEDI with a cross-direction autoregressive (AR) model and achieves better performance upon NEDI. In [3] [4], interpolation reconstructs image edges and isophotes iteratively with smooth constraint.

These methods have a limitation that they can only deal with enlargement whose scaling factor is  $2^i$ , ( $i = 1, 2, \dots$ ). But general scale enlargement and interpolation are required in many scenarios in reality. Wu *et al.* [5] proposed an adaptive resolution up-conversion method based on a two directional AR models implemented in H.264/SVC. Li *et al.* [6] constructed AR models with pixels neighbors instead of their available LR neighbors. The similarity between pixels within a local window is employed to depict the piecewise-stationarity of image signals.

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The methods mentioned above embody the neighborhood and edge information in an optimized function. Another type of methods identify the edge or isophote direction and interpolate along the direction, such as Wang's method [7] and segment adaptive gradient angle interpolation (SAGA) [8]. In [7], the isophote direction is estimated. Then it interpolates along a parallelogram which one edge of is parallel with the estimated isophote. SAGA models the isophote with a parameter called *displacement coefficient* and interpolates along isophote lattice rather than image lattice. By interpolating along the isophote lattice, these methods achieve desirable performance. And fractional locations can be represented in the isophote lattice so the general scale enlargement is supported.

However, two issues are not considered in these methods. First, when an interpolated line (a row or a column) is piecewise-stationary rather than global-stationary, the isophote estimation is inaccurate, inconsistent and varying dramatically, which causes undesirable interpolation results. Secondly, the edge or isophote direction is estimated by gradient. And the gradient calculation is sensitive to noises which remain widespread in natural images. Thus the isophote estimated by gradient is easy to deviate from the reality.

To overcome the obstacles mentioned above, we propose an interpolation algorithm based on fine-grained isophote model with consistency constraint. The algorithm employs the framework of isophote-based interpolation. First, the displacement coefficient is used to model the isophote. Fine-grained pixel intensity information is introduced to correct gradient-based displacement and make the isophote estimation more robust. Then, a gradient interpolation method along the isophote lattice with consistency constraint is proposed to force the direction of isophote consistent in the local window when the interpolated line is piecewise-stationary. Finally, An adaptive weighting fusion is employed to effectively fuse HR candidates from a HR pool. The proposed algorithm can accommodate the general scale enlargement. Experimental results show that our method achieves better performance, especially for regions including piecewise-stationary lines, than other methods.

The rest of the paper is organized as follows: Section 2 reviews the isophote-oriented interpolation algorithms. The proposed interpolation algorithm based on fine-grained isophote model with consistency constraint is presented in Section 3. Experimental results are presented in Section 4. Finally, Section 5 concludes the paper.

## 2. ISOPHOTE-ORIENTED INTERPOLATION

Conventional interpolations generate HR pixels along the image lattice, resulting in unnatural representation of edges. In order to depict the non-stationarity between two regions and keep interpolated edges natural, we introduce *isophote*. *Isophote* is a constant intensity line. Stationarity are maintained along the isophote and deviated in its vertical direction. Isophote-oriented interpolation calculates isophotes of images and interpolates along isophotes rather than image lattice to predict HR pixels based on similar stationary regions.

Before generating HR pixel values, an isophote coordinate is established at first. According to the definition of the isophote, we represent an isophote across two adjacent row  $i$  and  $(i + 1)$  as:

$$I(i, j) = I(i + 1, j + \alpha), \quad (1)$$

where  $I(i, j)$  is the intensity of LR pixel located in coordinate  $(i, j)$  and  $\alpha$  is called *displacement coefficient* indicating the location where the isophote passing through location  $(i, j)$  crosses with row  $(i + 1)$  as shown in Fig.2. When the isophote is approximated to a straight line, we can deduce Eqn.(1) with the first-order Taylor expansion:

$$I(i + 1, j + \alpha) = I(i, j) + I_x(i, j) + I_y(i, j) \cdot \alpha, \quad (2)$$

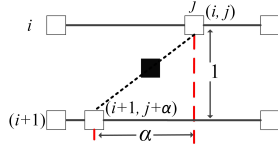
Given Eqn.(1), Eqn.(2) reduces to:

$$I_x(i, j) + I_y(i, j) \cdot \alpha = 0, \quad (3)$$

where  $I_x(i, j)$  and  $I_y(i, j)$  are the first-order derivatives of the intensity at location  $(i, j)$ .  $\alpha$  is deduced from Eqn.(4):

$$\alpha = -I_x(i, j) / I_y(i, j), \quad (4)$$

and the straight line between  $(i, j)$  and  $(i + 1, j + \alpha)$  determines the isophote cross  $(i, j)$ . With row  $i$  and  $(i + 1)$ , all these isophotes between LR pixels in row  $i$  like  $(i, j)$  and connected locations in row  $(i + 1)$  like  $(i + 1, j + \alpha)$  make up the *isophote lattice* between row  $i$  and  $(i + 1)$ .



**Fig. 2:**  $(i, j)$  and  $(i + 1, j + \alpha)$  connect into a isophote. The HR pixel in the isophote is determined by  $I(i, j)$  and  $I(i + 1, j + \alpha)$ .

Interpolation can be performed in the isophote lattice when the isophote exactly goes through the HR pixel to be interpolated and two nearby LR pixels as shown in Fig.2. However, in most cases the isophote does not intersect with the image lattice. For interpolation, Data in the image lattice must be projected to the isophote lattice and mapped back to the image lattice when the interpolation along the isophote is completed. In [7], a directional bilinear interpolation is employed in a parallelogram window consisting of a line parallel with local isophote. In SAGA [8], one dimensional interpolation is applied to grid LR data to the isophote lattice at first and then linear interpolation is employed along the isophote to calculate data value at the “match” location in HR row. Finally one dimensional method is applied again to gridded the data back to the high resolution lattice.

However, two issues are not considered in previous methods. First, piecewise-stationary lines cause inconsistent  $\alpha$ , which causes undesirable results. Second,  $\alpha$  is determined by  $I_x$  and  $I_y$  which are easy to be effected by noises, leading to the isophote estimated deviating from the reality.

### 3. INTERPOLATION BASED ON FINE-GRAINED ISOPHOTE MODEL WITH CONSISTENCY CONSTRAINT

In order to make the isophote estimation more robust to noises and interpolate in the condition of piecewise-stationary lines, we propose an interpolation algorithm based on fine-grained isophote model with consistency constraint. The proposed algorithm applies a two-layer displacement calculation to make the displacement estimation accurate in fine-grained and a gradient interpolation with consistency constraint to make the displacement estimation consistent in both global-stationary condition or piecewise-stationary condition. Fig.1 shows the entire work flow of the proposed method including five parts:

- **Preprocessing:** In order to make use of the isophote information in directions, the input LR images are converted to LR

candidates by simple conversions, such as upside down, transpose. The corresponding reverse conversions are performed to HR image candidates before fusion.

- **1-D interpolation:** HR pixels located in the line consisting of LR pixels is generated by one dimension interpolation.
- **Two-layer displacement calculation:** The gradient-based displacement is applied and the local window search displacement is put forward to introduce the fine-grained pixel intensity information to correct gradient-based displacement.
- **Gradient interpolation with consistency constraint:** A segment gradient interpolation is employed when an interpolated line is stationary while a consistent gradient interpolation is applied to overcome the inconsistent displacement when an interpolated line is piecewise-stationary.
- **Adaptive weighting fusion:** Adaptive weighting fusion generates HR patches in a fixed non-overlapped window. We utilize the weighting function in the form of formula in [9] to lower the weight of patches which contains artifacts or noises.

To further explain, we will employ the two-layer displacement calculation, gradient interpolation with consistency constraint and adaptive weighting fusion elaborated in the following subsections.

#### 3.1. Two-Layer Displacement Calculation

Gradient is a basic way to calculate the displacement and in most cases the gradient-based displacement reflects isophotes in reality. But in some cases, like in Fig.3, the gradient-based displacement is affected by noises or large deviations in the previous row  $R_{i-1}$  and deviates from real isophotes. The local window search displacement is proposed to compensate and correct the detail of the isophote according to fine-grained pixel intensity information. The local window search displacement is independent with any operator and only related with the specified LR pixel  $L(i, j)$  and the  $(i + 1)$ -th row  $R_{i+1}$ . Therefore the impact caused by the  $(i - 1)$ -th row  $R_{i-1}$  can be alleviated and the fine detail of the isophote can be preserved. Therefore a two-layer displacement calculation is proposed, including the gradient-based displacement ( $\alpha_1$ ) calculated as in [8] and the local window search displacement ( $\alpha_2$ ).

The approach of calculating  $\alpha_2(i, j)$  is to find a location in  $R_{i+1}$  where the pixel value is or estimated to be the same as the specified pixel  $L(i, j)$ . Two cases are considered to fulfil the goal.

- Case 1 (see in Fig.4(a)): Define  $k$  indicating a location in  $R_{i+1}$ . Search two adjacent LR pixels  $L(i + 1, k)$  and  $L(i + 1, k + 1)$  in  $R_{i+1}$  from center to both sides such that:

$$\begin{aligned} M_1 &= \max(L(i + 1, k), L(i + 1, k + 1)) > L(i, j), \\ M_2 &= \min(L(i + 1, k), L(i + 1, k + 1)) < L(i, j), \end{aligned} \quad (5)$$

Where  $k$  satisfies  $j - W_s < k < j + W_s - 1$  and  $W_s$  is the size of the search window. If two pixels are found,  $\alpha_2$  is calculated as:

$$\alpha_2 = \begin{cases} k - j + 1 - (L(i, j) - M_2) / (M_1 - M_2), & \text{if } L(i + 1, k) > L(i + 1, k + 1); \\ k - j - (L(i, j) - M_2) / (M_1 - M_2), & \text{if } L(i + 1, k) < L(i + 1, k + 1). \end{cases} \quad (6)$$

- Case 2 (see in Fig.4(b)): If there does not exist adjacent pixels in the local search window meeting the required condition, the pixel  $L(i + 1, k)$  whose value is most close to  $L(i, j)$  is regarded as locating in the same isophote with  $L(i, j)$ . And  $\alpha_2$  can be calculated as:

$$\alpha_2 = k - j. \quad (7)$$

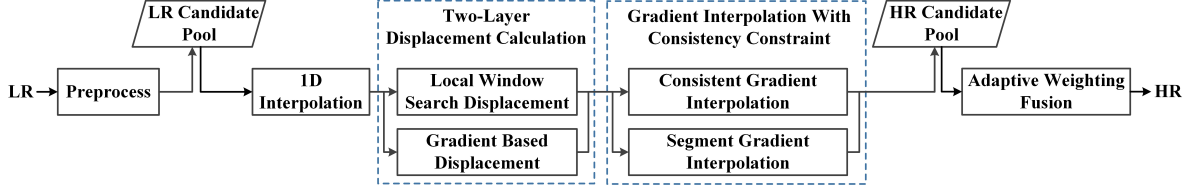


Fig. 1: Flow diagram of the proposed method based on fine-grained isophote model with consistency constraint.

When  $\alpha_1$  and  $\alpha_2$  are calculated, we simply combine them by a linear summation:

$$\bar{\alpha} = (1 - \gamma)\alpha_1 + \gamma\alpha_2, \quad (8)$$

$\gamma$  balances the weight to control the importance of two items.

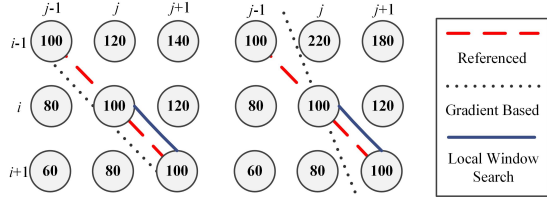


Fig. 3: The large deviation in  $R_{i-1}$  distorts the direction of the isophote in the gradient-based displacement method while the local window search displacement can avoid such distortion.

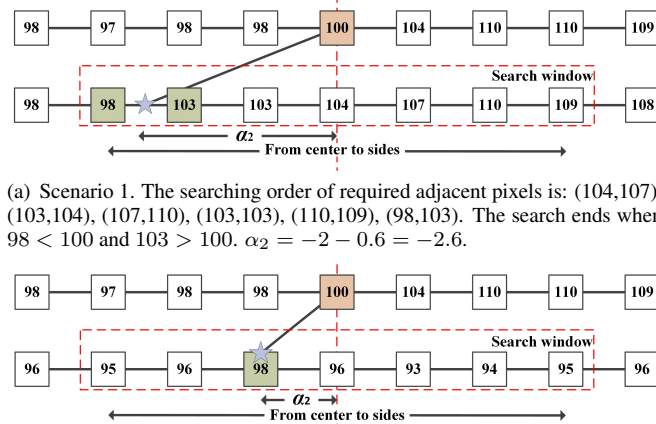


Fig. 4: Examples of local window search displacement.

### 3.2. Gradient Interpolation With Consistency Constraint

The displacement  $\bar{\alpha}$  is used to establish the isophote lattice in the gradient interpolation with consistency constraint.

When an interpolated line is piecewise-stationary, the displacements calculated is inconsistent and varies dramatically. Large varying displacements causes undesirable performance of the segment gradient interpolation. However, in most cases the real displacements are stationary and consistent in the local even in the condition that an interpolated line is piecewise-stationary rather than global-stationary. So for piecewise-stationary lines, displacements need to be handled cautiously to keep consistent in interpolation.

By considering whether an interpolated line is global-stationary or piecewise-stationary, the gradient interpolation with consistency constraint chooses an appropriate interpolation method adaptively. The segment gradient interpolation in [8] is adopted when the line is global-stationary and the consistent gradient interpolation is applied when the line is piecewise-stationary. The consistent gradient interpolation mitigates the problem without sacrificing the fine detail of

the isophote by keeping the displacements of all pixels consistent in a local window for each interpolated pixel.

The interpolation is performed in windows and one HR value is generated once. When interpolating  $H(i, j)$ , we firstly determine its corresponding LR window (consisting of  $L(m, n)$ ,  $L(m + 1, n)$ ,  $L(m, n + 1)$  and  $L(m + 1, n + 1)$ ).  $\bar{\alpha}(m, n)$ , the displacement coefficient of  $L(m, n)$  (LR pixel in the upper left corner of LR window), is chosen as the displacement value of the corresponding pixel  $L(m, n)$  and five nearby LR pixels in the same row ( $L(m, n - 2)$ ,  $L(m, n - 1)$ ,  $L(m, n + 1)$ ,  $L(m, n + 2)$ ,  $L(m, n + 3)$ ).  $\bar{\alpha}$  in the interpolation window are set to be consistent and all six LR pixels' isophote connected locations in the next row is decided by the same  $\bar{\alpha}(m, n)$  instead of their own displacement values. Using the same displacement value only related to the interpolated pixel in the LR window guarantees both consistency and fidelity of the evaluated isophote, as shown in Fig.5. Then the grid projection method in [8] is employed.

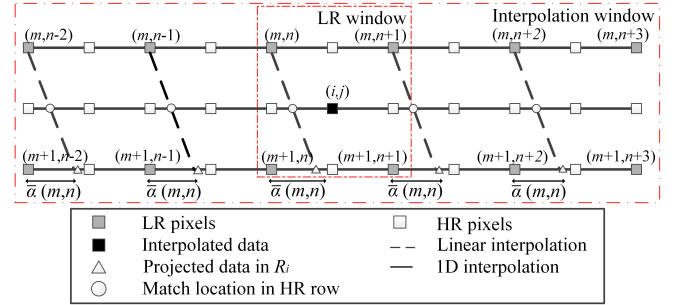


Fig. 5: The consistent gradient interpolation determines isophote lattices according to interpolated pixels'  $\alpha$  in concordance to keep the isophote estimation both consistency and fidelity.

### 3.3. Adaptive Weighting Fusion

After the gradient interpolation with consistency constraint, we get HR candidates in the candidate pool. Then two intuitions are considered to guide the fusion of HR candidates. First, the performances of different interpolations (such as one dimensional interpolation or the gradient interpolation) are different and the weights given to pixels from different interpolations should be handled respectively. Second, the region-based fusion provides more information than the pixel-based fusion and fusion performing in region are better than in pixel. Based on these, we design an adaptive weighting fusion method. First, other than preserving LR pixels exactly locating on the HR lattice, we split HR pixels in HR candidates to clusters by considering their performances. For example, we group the pixels from exactly 1-D interpolation or group the pixels from combination of 1-D interpolation and the gradient interpolation. Second, Fusion is deployed in patch. In order to remove the outlier or noises, we employ a weighting function  $\exp(-x^2 / c)$  in [9]. Define  $P_i$  ( $i \in 1, 2, 3, 4$ ) as the local patch of intermediate results in the same region and  $\bar{P}$  as the average estimate for the high resolution patch:

$$\bar{P} = \frac{1}{4} \sum_{i=1}^4 P_i. \quad (9)$$

Define  $w_i$  as the weight of  $P_i$ , the high resolution patch is calculated according to:

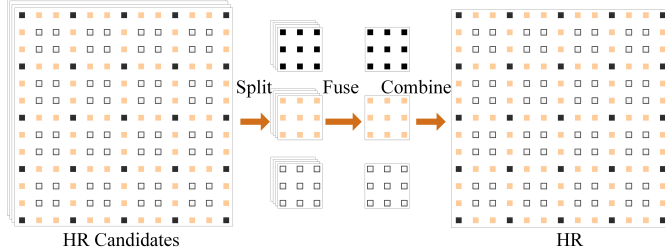
$$P(m, n) = \frac{\sum_{i=1}^4 w_i(m, n) P_i(m, n)}{\sum_{i=1}^4 w_i(m, n)}, \quad (10)$$

where the weighting function  $w_i$  is defined as:

$$w_i = \exp\left(-\frac{\Delta_i}{c}\right), \quad (11)$$

$$\Delta_i = \sum_{m, n} \left\| \frac{4}{3} \bar{P}(m, n) - P_i(m, n) \right\|_2, \quad (12)$$

where  $c$  is called cliff coefficient, distinguishing whether a value is a noise or outlier. Finally clusters are combined back to a whole image. The procedure is shown in Fig.6.



**Fig. 6:** An example of performance-based split / combine method in adaptive weighting fusion when performing  $3 \times$  interpolation

#### 4. EXPERIMENTAL RESULTS

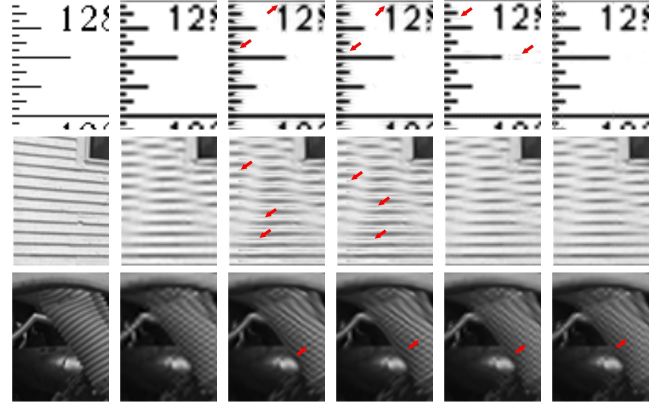
The proposed interpolation is implemented on MATLAB 8.2 platform. It is compared with the conventional Bicubic interpolation method and three state-of-the-art interpolation methods: NEDI [1], SAI [2] and BSAGA. We implemented BSAGA, which is an improved algorithm on SAGA [8]. It performs bicubic interpolation where piecewise-stationary lines appear. BSAGA avoids majority of stripe-like artifacts caused by large varying displacement coefficient and performs better than SAGA. We have tested these interpolation algorithms on a large numbers of images including Kodak database and other standard testing images.

To generate LR images, a bicubic down-sampling is performed to original HR images by a factor of  $1/s$  and  $s$  is the scaling factor. Then we use different interpolation methods to generate HR images. The results are compared with original HR images. Peak Signal-to-Noise Ratio (PSNR) is chosen as the evaluating criterion and  $s = 2$  in objective evaluation.

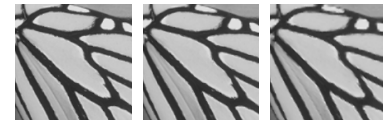
**Table 1:** Average PSNR (dB) of results in different methods, the scaling factor  $s = 2$

Images	Bicubic	NEDI	SAI	BSAGA	Proposed
<i>Child</i>	35.49	34.56	<b>35.63</b>	35.41	35.48
<i>Cameraman</i>	25.51	25.44	25.99	25.98	<b>26.02</b>
<i>Monarch</i>	31.93	31.80	<b>33.08</b>	32.52	32.58
<i>Airplane</i>	29.40	28.00	29.62	29.72	<b>29.84</b>
<i>Statue</i>	31.36	31.01	31.78	31.75	<b>31.81</b>
<i>Lighthouse</i>	26.97	26.37	26.70	27.23	<b>27.25</b>
<i>Barbara</i>	<b>24.46</b>	22.36	23.55	24.24	24.23
Average	29.30	28.51	29.48	29.55	<b>29.61</b>

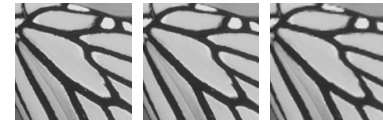
Weight coefficients  $\gamma$ ,  $c$  are set as 0.06, 3000. Size of a patch in fusion step is set to be  $5 \times 5$ . Larger size of window leads to the weight of each HR candidate turning similar in different windows while smaller size may result in lacking of enough region information. Results of interpolations are shown in Table 1. It shows that, the proposed method produces comparable or better PSNR results than other methods. The average PSNR of the proposed method gains 0.13dB and 0.06dB over SAI and BSAGA. It is worth noticing that for *Babara* and *Lighthouse*, the proposed method gains 0.68 dB



**Fig. 7:** Visual comparisons: Portions from various interpolated images using different methods. From top to bottom: *ruler*, *lighthouse*, *bike*. From left to right: ground truth, Bicubic, NEDI, SAI, BSAGA, the proposed method.



(a) 35.63(dB) (b) 33.95(dB) (c) 29.86(dB)



(d) 35.73(dB) (e) 34.03(dB) (f) 29.90(dB)

**Fig. 8:** Visual comparisons: Portions from various scaling factor  $s$  using different methods. Test image: *Monarch*. From top to bottom: BSAGA, the proposed method. From left to right:  $s = 1.5, 1.7, 2.5$ .

and 0.55 dB respectively over SAI. For *Airplane* and *Statue*, the proposed method gains 0.12 dB and 0.10 dB respectively over BSAGA.

We also compare the visual quality of the different interpolation methods as shown in Fig.7. Bicubic interpolation blurs the edge. The edge-directed methods like NEDI and SAI preserve the long edge structure well. However, these methods produce annoying artifacts like fake short edges nearby the fast-evolving edges. BSAGA produces better results, but stripe-like artifacts still occur in piecewise-stationary lines. The proposed method produces smaller interpolation errors than other methods, especially on the regions pointed by the red arrows. General scaling results interpolated by the proposed method is shown in Fig.8 and the PSNR of the proposed method gains over BSAGA. For more experimental results, please visit our project website: <http://www.icst.pku.edu.cn/course/icb/isophoteInterp.html>.

#### 5. CONCLUSION

In this paper, we propose a general scale interpolation algorithm based on fine-grained isophote model with consistency constraint. The proposed model characterizes the piecewise-stationarity of image signals. To handle the piecewise-stationarity, we force the direction of the isophote consistent in the local window when an interpolated line is piecewise-stationary. The fine-grained pixel intensity is introduced to correct the displacement calculation and make the isophote estimation more robust. Experimental results demonstrate the proposed approach achieves better performance in both objective and subjective quality assessment.

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